M.Sc. (Mathematics) (NEP Pattern) Semester-I

NEP-64-3 - Ordinary Differential Equations

P. Pages : 3 **GUG/S/25/15117**

Time : Three Hours Max. Marks : 80

* 2 7 6 1 *

Notes : 1. Solve all five questions.
2. Each questions carries equal marks.

UNIT - I

1. a) Consider the equation $y' + ay = b(x)$, where a is a constant and b is a continuous function on an interval. If x_0 is a point in I and C is any constant, then prove that the function ϕ defined by $\phi(x) = e^{-ax} \int_{x_0}^{x} e^{at} b(t) dt + ce^{-ax}$ is a solution of this equation. Also, show that every solution has this form. **8**

b) Suppose a and b are continuous functions on an interval I let A be a function such that $A' = a$. Then the function ψ is given by $\psi(x) = e^{-A(x)} \int_{x_0}^{x} e^{A(t)} b(+) dt$, where x_0 is in I, is a solution of the equation $y' + a(x)y = b(x)$ on I. The function ϕ_1 given by $\phi_1(x) = e^{-A(x)}$ is a solution of the homogeneous equation $y' + a(x)y = 0$. If C is any constant, $\phi = \psi + c\phi_1$ is a solution of $y' + a(x)y = b(x)$ and every solution of $y' + a(x)y = b(x)$ has this form. **8**

OR

c) Consider the equation $y' + ay = 0$, where a is a complex constant If is any complex number, the function ϕ defined by $\phi(x) = ce^{-ax}$ is a solution of this equation, and moreover every solution has this form. **8**

d) Let ϕ_1, ϕ_2 be the two solutions of $L(y) = 0$ given by $\phi_1(x) = e^{r_1 x}$, $\phi_2(x) = e^{r_2 x}$ in case $r_1 \neq r_2$ and by $\phi_1(x) = e^{r_1 x}$, $\phi_2(x) = xe^{r_1 x}$ in case $r_1 = r_2$. If c_1, c_2 are any two constants the function $\phi = c_1 \phi_1 + c_2 \phi_2$ is a solution of $L(y) = 0$ on $-\infty < x < \infty$ conversely, if ϕ is any solution of $L(y) = 0$ on $-\infty < x < \infty$, there are unique constant c_1, c_2 such that $\phi = c_1 \psi_1 + c_2 \phi_2$. **8**

UNIT - II

2. a) State and prove Existence Theorem. **8**

b) Let $\phi_1,....,\phi_n$ be n solutions of $L(y)=0$ on an interval I, and let x_0 be any point in I. **8**

Then prove that $W(\phi_1,....,\phi_n)(x)=\exp\left[-\int_{x_0}^{x} a_1(t)dt\right]W(\phi_1,.....,\phi_n)(x_0)$

OR

c) State and prove Existence Theorem for Analytic Coefficients. **8**

d) Solve the Eulers equation of second order. **8**

UNIT - III

3. a) Let g, h be continuous real-valued functions for $a \le x \le b, c \leqq y \leqq d$ respectively, and consider the equation $h(y)y' = g(x)$. If G, H are any functions such that $G' = g, H' = h$, and C is any constant such that the relation $H(y) = G(x) + C_1$ define a real-valued differential function ϕ for x in some interval I contained in $0 \leqq x \leqq b$ then prove that ϕ will be a solution of $h(y)y' = g(x)$ on I. Conversely, If ϕ is a solution of equation on I, it satisfies the relation $H(y) = G(x) + C$ on I, for some constant C. **8**

b) Let M, N be two real-valued functions which have continuous first partial derivative on some rectangle $R : |x - x_0| \le a$, $|y - y_0| \leqq b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if, and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R. **8**

OR

c) A function ϕ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)dt$ on I. **8**

d) Let F be continuous and satisfy a Lipschitz condition on R If ϕ and ψ are two solutions of $y' = f(x, y), y(x_0) = y_0$, on an interval I containing x_0, then prove that $\phi(x) = \psi(x)$ for all x in I. **8**

UNIT - IV

4. a) Explain some special equations. **8**

b) Suppose F is a vector-valued function defined for (x, y) on a set S of the form $|x - x_0| \leqq a, |y - y_0| \leqq b, (a, b > 0)$, an of the form $|x - x_0| \leqq a, |y| < \infty, (a > 0)$. If $\frac{\partial f}{\partial y_k}(k = 1,....,n)$ exists, is continuous on S, and there is a constant $k > 0$ such that $\left|\frac{\partial f}{\partial y_k}(x, y)\right| \leqq k\ (k = 1,....,n) \forall x, y \in S$, then prove that f satisfies a Lipschitz condition on S with Lipchitz constant K. **8**

OR

c) Let f be a continuous vector-valued function defined on $R : |x - x_0| \leqq a,\ |y - y_0| \leqq b,\ (a, b > 0)$, and suppose F satisfies a Lipchitz condition on R. If M is a constant such that $|f(x, y)| \leqq M$ for all (x, y) in R, the successive approximations $\{\phi_k\}, (k = 0, 1, 2, ...)$, given by $\phi_0(x) = y_0$ converges on the interval $I : |x - x_0| \leqq \propto =$ minimum $\{a, b/m\}$, to a solution ϕ of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on I. **8**

d) Let $a_1, a_2, ..., a_n, b$ be continuous complex-valued functions on an interval I containing a point x_0. If $\alpha_1, \alpha_2, ..., \alpha_n$ are any n constants, there exists one, and only one, solution of the equations. **8**

$y^{(n)} + a_1(x) y^{(n-1)} + ... + a_n(x) y = b(x)$ on I satisfying

$\phi(x_0) = \alpha_1, \phi^1(x_0) = \alpha_2, ..., \phi^{(n-1)}(x_0) = \alpha_n$.

5. Solve the following.

a) Define linear dependence and independence. **4**

b) Prove that there exists n linearly independent solution of $L(y) = 0$ on I. **4**

c) Find all real-valued solution ϕ of $y' = \dfrac{x + x^2}{y - y^2}$. **4**

d) Solve the equation **4**

$y'' + k^2 y = 0, k > 0$

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